1.1 Functions

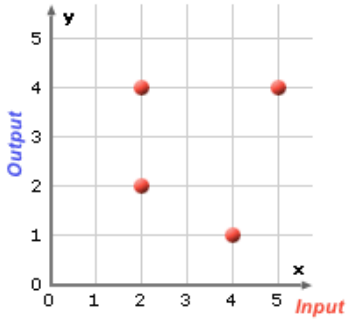
A Relations

A (binary) relation is defined as a set of ordered pairs (x, y).

A relation can be described using:

- words
- graphs
- equations
- inequalities
- sets of ordered pairs
- mapping diagrams

Ex 1. A relation is given by its graph as shown in the figure below. Write the relation as a set of ordered pairs.

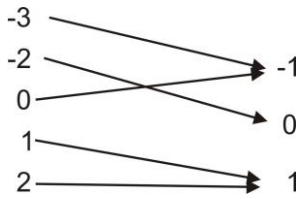


B Domain and Range of a Relation

The domain of the relation is the set of all the x values such that the ordered pair (x, y) satisfies the relation (is an element of the relation).

The range of the relation is the set of all the y values such that the ordered pair (x, y) satisfies the relation (is an element of the relation).

Ex 2. Find the domain and the range of the relation defined by the following mapping diagram:



C Functions

A function from a set X (called the domain) to a set Y (called the range) is a rule that assign to each element $x \in X$ exactly one element $y \in Y$ ($f : X \to Y$).

Use the function notation to represent the correspondence:

$$y = f(x)$$

- x is called the argument or the input of the function
- y is called the value or the output of the function

Reading: "f of x" or "f at x"

Ex 3. Consider the function $f(x) = (x-1)^2$. Find:

a) $f(0)$

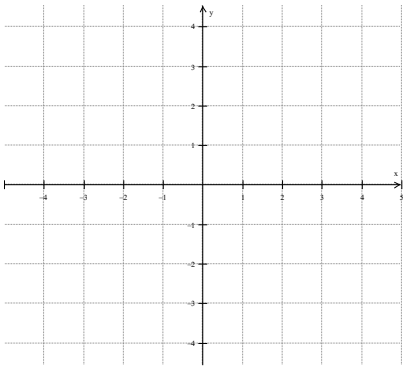
b) $f\left(\frac{1}{2}\right)$

c) $f(a+2)$

D Graph

The graph of a function f is the graph of the set of ordered pairs (x, y) where $y = f(x)$.

Ex 4. Graph the function defined by a set of ordered pairs: $f = \{(2,3),(0,-2),(-4,3),(4,0),(-3,-3)\}$.



E The Vertical Line Test

Any function is a relation but not all relations are functions.

A graph represents a function if every vertical line intersects the graph in at most one point.

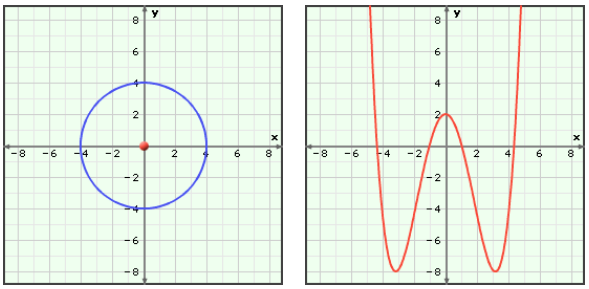
Ex 5. For each case, verify if the set of ordered pairs represents or does not represent a function.

a) $f = \{(0,0),(-1,-1),(2,2),(1,-1)\}$

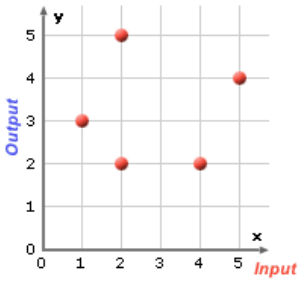
b) $f = \{(2,3),(-1,3),(2,-2),(-3,-1)\}$

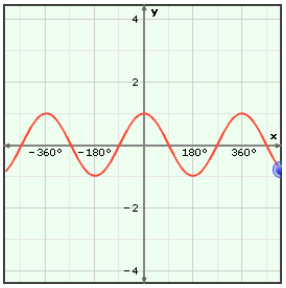
Ex 6. For each case, verify if the graph represents or does not represent a function.

a) b)



c) d)





F Domain and Range

The domain D of a function f is the set of all real numbers x for which $y = f(x)$ is defined.

The range R of a function f is the set of all real numbers y for which $y = f(x)$ is defined.

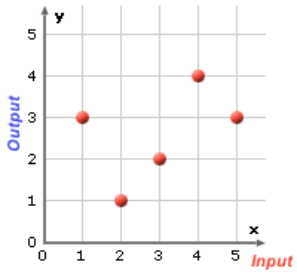
Ex 7. Find the domain and the range of each function defined by a set of ordered pairs.

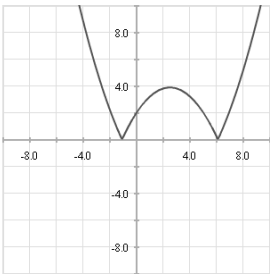
a) $f = \{(-2,0),(-1,1),(0,-1),(1,0)\}$

b) $f = \{(-1,0),(0,1),(1,0),(3,1),(7,0)\}$

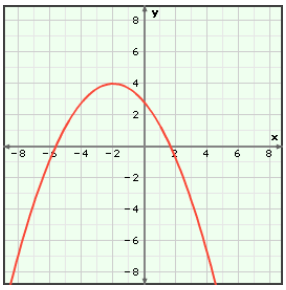
Example 8. Find the domain and the range of each function defined by its graph.

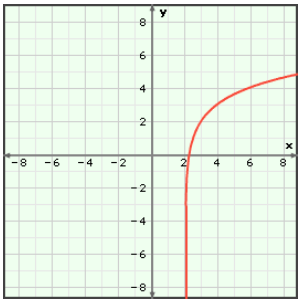
a) b)





c) d)





G Restrictions

Division by 0 is not defined. So:

$$denominator \neq 0$$

Square root is defined for a non negative number. So:

$$\sqrt{x}; \quad x \geq 0$$

A square is not a negative number. So:

$$x^2 \geq 0$$

A square root is not a negative number. So:

$$\sqrt{x} \geq 0$$

Ex 9. Use the restrictions to find the domain and the range of each function defined by a formula.

a) $y = (x-1)^2 - 3$

b) $y = 2 + \sqrt{x-3}$

c) $y = \dfrac{x-2}{x+2}$

Reading: Nelson Textbook, Pages 4-10

Homework: Nelson Textbook, Page 11: #1, 2, 3, 4, 5, 8, 10, 12, 14, 15